EXERCISE – III HINTS & SOLUTIONS

Sol.1 $a + 2d = 18$
$a + 6d = 30 \quad \Rightarrow 4d = 12$
$\Rightarrow d = 3 \ \& \ a = 12$

$S_{17} = \frac{17}{2}[24 + 16 \cdot 3] = 17 [12 + 24] = 612$

Sol.2 between 100 to 1000 $\Rightarrow$ (100, 1000)
(i) divisible by 7
$a = 105, d = 7, \ell = 994$
$994 = 105 + (n - 1) 7 \Rightarrow n = 128$
(ii) not divisible by 7
$999 - 100 = 899$
$= 899 - 128 = 771$

Sol.3 no. (100, 800)
$(16k + 7) \in (100, 800) \qquad k \in N$
First no. $= 103 = a = 16 \times 6 + 7 = 96 + 7$
Last no. is $= 784 + 7 = 791 = \ell = 16 (49) + 7$
$n = 49 - 5 - 44$

$S_n = \frac{44}{2}[103 + 791] = 22 \times 894 = 19668$

Sol.4 $(a - d) + a + (a + d) = 27$
$3a = 27 \Rightarrow a = 9$
$\& \ a (a^2 - d^2) = 504$
$81 - d^2 = 56 \Rightarrow d^2 = 25$
$\Rightarrow d = \pm 5$
$9 + 5, 9, 9 - 5 \Rightarrow 14, 9, 4$
or $9 - 5, 9, 9 + 5 \Rightarrow 4, 9, 14$

Sol.5 a, b, c in A.P.
(i) $a^2 (b + c), b^2 (c + a), c^2 (a + b)$ in A.P.
$\Rightarrow a^2 (b + c) + abc, b^2 (c + a) + abc, c^2 (a + b) + abc$, in A.P.
$\Rightarrow a (ab + ac + bc), b (bc + ab + ac), c(ca + bc + ab)$, in A.P.
$\Rightarrow$ a, b, c in A.P.
(ii) a, b, c in A.P.
$-2a, -2b, -2c$ in A.P.
$(a + b + c - 2a), (a + b + c - 2b), (a + b + c) - 2c)$ in A.P.
$(b + c - a), (c + a - b), (a + b - c)$ in A.P.

Sol.6 $\frac{a}{r} . a. ar = 216 \Rightarrow a^3 = 216 \Rightarrow a = 6$

$\frac{a}{r} a + a. ar + \frac{a}{r}.ar = 156$

$\Rightarrow 6\left(\frac{1}{r}+r+1\right) = 26 \qquad \Rightarrow 3\left(\frac{1}{r}+r+1\right) = 13$

$\Rightarrow 3 + 3r^2 + 3r = 13r \qquad \Rightarrow 3r^2 - 10r + 3 = 0$

$\Rightarrow (r - 3) (3r - 1) = 0 \Rightarrow r = 3$, or $r = \frac{1}{3}$

$r = 3 \qquad \Rightarrow 2, 6, 18$

or $r = \frac{1}{3} \qquad \Rightarrow 18, 6, 2$

Sol.7 $T_p = AR^{p-1} = a$
$T_q = AR^{q-1} = b$
$T_r = AR^{r-1} = c$

$a^{q-r} . b^{r-p} . c^{p-q} = \left(\frac{a}{c}\right)^q \left(\frac{b}{a}\right)^r \left(\frac{c}{b}\right)^p$

$= (R^{p-r})^q (R^{q-p})^r (R^{r-q})^p = R^0 = 1$

Sol.8 $(a - d) + a + (a + d) = 21$
$\Rightarrow 3a = 21 \qquad \Rightarrow a = 7$
$\Rightarrow (a - d), (a - 1), (a + d + 1)$ in G.P.
$(7 - d), 6, (8 + d)$ in G.P.
$36 = (7 - d) (8 + d)$
$\Rightarrow d^2 + d - 20 = 0 \Rightarrow (d + 5) (d - 4) = 0$
$\Rightarrow d = -5$ or $d = 4$
If $d = -5 \Rightarrow 12, 7, 2$ or $d = 4 \Rightarrow 3, 7, 11$

Sol.9 $a + ar + ar^2 + ... \infty = 4 \Rightarrow \frac{a}{1-r} = 4$

$a^3 + a^3r^3 + a^3r^6 + ... \infty = 192 \Rightarrow \frac{a^3}{1-r^3} = 192$

$\frac{4^3(1-r)^3}{(1-r^3)} = 192 \Rightarrow \frac{(1-r)^2}{(1+r+r^2)} = 3$

$\Rightarrow 1 - 2r + r^2 = 3 + 3r + 3r^2$
$\Rightarrow 2r^2 + 5r + 2 = 0$
$\Rightarrow (r + 2) (2r + 1) = 0 \qquad \Rightarrow r = -2$ reject

$\Rightarrow r = -\frac{1}{2} \qquad \because |-2| \not< 1$

$r = -\frac{1}{2}, a = 6 \qquad \Rightarrow 6, -3, \frac{3}{2}, -\frac{3}{4}, ...$

Sol.10 (i) A.G.P. with $r = \frac{1}{2}$

$$S = 1 + \frac{2}{2} + \frac{3}{2^2} + \frac{4}{2^3} + ... + \frac{n}{2^{n-1}}$$

$$\frac{S}{2} = \frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + ... + \frac{n-1}{2^{n-1}} + \frac{n}{2^n}$$

$$\Rightarrow \frac{S}{2} = 1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + ... + \frac{1}{2^{n-1}} - \frac{n}{2^n}$$

$$\Rightarrow S = 2 + 1 + \frac{1}{2} + \frac{1}{2^2} + ... + \frac{1}{2^{n-2}} - \frac{n}{2^{n-1}}$$

$$\Rightarrow S = 2 + \left(1 + \frac{1}{2} + \frac{1}{2^2} + ... + \frac{1}{2^{n-2}}\right) - \frac{n}{2^{n-1}}$$

$$= 2 + \frac{\left(1 - \left(\frac{1}{2}\right)^{n-1}\right)}{1 - \frac{1}{2}} - \frac{n}{2^{n-1}}$$

$$= 2 + 2\left(1 - \frac{1}{2^{n-1}}\right) - \frac{n}{2^{n-1}} = 4 - \frac{(n+2)}{2^{n-1}}$$

(ii) $T_n = \frac{2^n - 1}{4^{n-1}} = \frac{2^n - 1}{2^{2n-2}} = \frac{2^n}{2^{2n-2}} - \frac{1}{2^{2n-2}}$

$$S = \sum_1^\infty T_n = \sum_1^\infty \frac{1}{2^{n-2}} - \sum_1^\infty \frac{1}{2^{2n-2}}$$

$$S = \left(2 + 1 + \frac{1}{2} + ...\infty\right) - \left(1 + \frac{1}{2^2} + \frac{1}{2^4} + ...\infty\right)$$

$$S = \frac{2}{1 - \frac{1}{2}} - \frac{1}{1 - \frac{1}{4}} = 4 - \frac{4}{3} = \frac{8}{3}$$

Sol.11 $\sum_{r=1}^{n} T_r = \sum_{r=1}^{n} (2r+1)\,2^r$

$S = 3.2 + 5.2^2 + 7.2^3 + ... + (2n + 1)\,2^n$

$2S = 3.2^2 + 5.2^3 + ...+ (2n–1)\,2^n + (2n+1)\,2^{n+1}$

$– S = 3.2 + 2.2^2 + 2.2^3 + ... + 2.2^n – (2n + 1)\,2^{n+1}$

$(–1)\,S = 6 + 2^3\,(1 + 2 + 2^2 + ... + 2^{n–2}) – (2n + 1)\,2^{n+1}$

$$(-1)\,S = 6 + 2^3\,\frac{(2^{n-1} - 1)}{2 - 1} - (2n + 1)\,2^{n+1}$$

$(–1)\,S = 6 + 2^{n+2} – 8 – (2n + 1)\,2^{n+1}$

$S = (2n + 1)\,2^{n+1} – 2^{n+2} + 2 = 2^{n+1}\,(2n – 1) + 2$

Sol.12 $T_7 = \frac{1}{a + 6d} = \frac{1}{20} \quad \Rightarrow a + 6d = 20$

$T_{13} = \frac{1}{a + 12d} = \frac{1}{38} \quad \Rightarrow a + 12d = 38$

$\Rightarrow 6d = 18 \quad \Rightarrow d = 3\ \&\ a = 2$

$T_4 = \frac{1}{a + 3d} = \frac{1}{2 + 9} = \frac{1}{11}$

Sol.13 $\frac{a+b}{2} = 6 \Rightarrow a + b = 12,\ G = \sqrt{ab},\ H = \frac{2ab}{a+b}$

$\&\ G^2 + 3H = 48$

$\therefore\ H = \frac{2G^2}{12} \quad \Rightarrow\ G^2 = 6H$

$\therefore\ 6H + 3H = 48 \Rightarrow\ H = \frac{16}{3}$

$\&\ G^2 = 32 = ab$

$\therefore\ a = 4\ b = 8$

Sol.14 (i) $0 < \theta < \frac{\pi}{2} \Rightarrow \cot\theta > 0\ \&\ \tan\theta > 0$

A.M. $\geq$ G.M. $\quad \Rightarrow \frac{\tan\theta + \cot\theta}{2} \geq (\tan\theta \cot\theta)^{1/2}$

$\Rightarrow\ \tan\theta + \cot\theta \geq 2$

(ii) A.M. > G.M. {x,y,z are different}

$$\frac{x^2y + y^2z + z^2x}{3} > (x^3y^3z^3)^{1/3}$$

$$\frac{xy^2 + yz^2 + zx^2}{3} > (x^3y^3z^3)^{1/3}$$

Multiply both

$(x^2y + y^2z + z^2x)\,(xy^2 + yz^2 + zx^2) > 9x^2y^2z^2$

(iii) A.M. $\geq$ G.M.

$$\frac{a+b}{2} \geq (ab)^{1/2}\ \&\ \frac{b+c}{2} \geq (bc)^{1/2}\ \&\ \frac{c+a}{2} \geq (ca)^{1/2}$$

multiply all

$\Rightarrow\ (a + b)\,(b + c)\,(c + a) \geq 8abc$

Sol.15 **(i)** $\sum_{n=1}^{n} T_n = \sum_{1}^{n} n(n+2) = \Sigma n^2 + 2\Sigma n$

$$= \frac{n(n+1)(2n+1)}{6} + \frac{2n(n+1)}{2}$$

$$= \frac{n(n+1)}{6}[2n+1+6] = \frac{n(n+1)(2n+7)}{6}$$

(ii) $\sum_{1}^{n} T_n = \sum_{1}^{n}(3^n - 2^n) = \sum_{1}^{n} 3^n - \sum_{1}^{n} 2^n$

$= (3 + 3^2 + ... + 3^n) - (2 + 2^2 + 2^3 + ... + 2^n)$

$$= \frac{3(3^n-1)}{3-1} - 2\frac{(2^n-1)}{2-1} = \frac{3}{2}(3^n-1) - 2(2^n-1)$$

$$= \frac{3^{n+1}}{2} - 2^{n+1} - \frac{3}{2} + 2 = \frac{1}{2}(3^{n+1}+1) - 2^{n+1}$$

Sol.16 $S_{10} = \frac{10}{2}[2a + 9d] = 155$

$\Rightarrow$ $2a + 9d = 31$

& G.P. have $d + da = 9 \Rightarrow d = \frac{9}{(a+1)}$

$\therefore$ $2a + \frac{81}{a+1} = 31$

$\Rightarrow$ $2a^2 + 2a + 81 = 31a + 31$

$\Rightarrow$ $2a^2 - 29a + 50 = 0$

$\Rightarrow$ $(2a - 25)(a - 2) = 0$

$$a = 2 \text{ or } a = \frac{25}{2}$$

If a = 2, d = 3

A.P. $\rightarrow$ 2, 5, 8, 11, ...

G.P. $\rightarrow$ 3, 6, 12, ...

If $a = \frac{25}{2}$, $d = \frac{.2}{3}$ A.P. $\rightarrow \frac{25}{2}, \frac{79}{6}, \frac{83}{6}, ...$

G.P. $\rightarrow \frac{2}{3}, \frac{25}{3}, \frac{625}{6}, ...$

Sol.17 **(i)** 1, (2, 3), (4, 5, 6, 7), (8, 9, ... , 15), ...

In n^{th} group = $\underbrace{(2^{n-1}, 2^{n-1}+1, 2^{n-1}+2, ...)}_{(2^{n-1})\text{term}}$

$$S_n = \frac{2^{n-1}}{2}[2.2^{n-1} + (2^{n-1} - 1) . 1]$$

$= 2^{n-2}[2^n + 2^{n-1} - 1]$

(ii) 1, (2, 3, 4), (5, 6, 7, 8, 9), ...

n^{th} group $\underbrace{(\text{.........} n^2-2, n^2-1, n^2)}_{(2n-1)\text{terms}}$

$\therefore$ $S_n = \frac{(2n-1)}{2}[2n^2 + (2n - 1 - 1)(-1)]$

$= \frac{(2n-1)}{2}[2n^2 - 2n + 2]$

$= (2n - 1)(n^2 - n + 1)$

$= 2n^3 - 3n^2 + 3n - 1 = n^3 + (n - 1)^3$

Sol.18 $S = \frac{5}{13} + \frac{55}{13^2} + \frac{555}{13^3} + \frac{5555}{13^4} + ... \infty$

$$S = \frac{5}{9}\left[\frac{9}{13} + \frac{99}{13^2} + \frac{999}{13^3} + \frac{9999}{13^4} + ...\infty\right]$$

$$S = \frac{5}{9}\left[\left(\frac{10}{13} + \left(\frac{10}{13}\right)^2 + \left(\frac{10}{13}\right)^3 + \left(\frac{10}{13}\right)^4 + ...\infty\right) - \left(\frac{1}{13} + \frac{1}{13^2} + \frac{1}{13^3} + \frac{1}{13^4} + ... - \infty\right)\right]$$

$$S = \frac{5}{9}\left[\frac{\frac{10}{13}}{1-\frac{10}{13}} - \frac{\frac{1}{13}}{1-\frac{1}{13}}\right] = \frac{5}{9}\left[\frac{10}{3} - \frac{1}{12}\right] = \frac{5}{9} \times \frac{39}{12} = \frac{65}{36}$$

Sol.19 $0 < x < \pi$

exp. $\{(1+|\cos x| + \cos^2 x + |\cos^3 x| + \cos^4 x + ... \infty)\log_e 4\}$

$$= \exp\left\{\frac{\log_e 4}{1-|\cos x|}\right\} = \left\{e^{\log_e 4}\right\}^{\frac{1}{1-|\cos x|}} = 4^{\frac{1}{1-|\cos x|}}$$

$y^2 - 20y + 64 = 0$

$\Rightarrow$ $(y - 16)(y - 4) = 0$ $\Rightarrow$ $y = 4$ or $y = 4^2$

$\therefore$ $4^{\frac{1}{1-|\cos x|}} = 4$ or $4^{\frac{1}{1-|\cos x|}} = 4^2$

$\frac{1}{1-|\cos x|} = 1$ $\qquad$ $\frac{1}{1-|\cos x|} = 2$

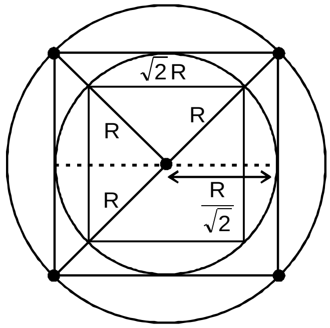
$1 - |\cos x| = 1$ $\qquad$ $1 - |\cos x| = \frac{1}{2}$

$\cos x = 0$ $\qquad$ $|\cos x| = \frac{1}{2}$

$x = \frac{\pi}{2}$ $\qquad$ $x = \frac{\pi}{3}, \frac{2\pi}{3}$

Sol.20 radius $\rightarrow$ R, $\frac{R}{\sqrt{2}}$, $\frac{R}{2}$, $\frac{R}{2\sqrt{2}}$ .... ∞

sides $\rightarrow$ $\sqrt{2}$ R, R, $\frac{R}{\sqrt{2}}$, $\frac{R}{2}$ ....∞



<u>Circle</u>

sum of areas = $\pi\left(R^2 + \frac{R^2}{2} + \frac{R^2}{4} + \frac{R^2}{8} .... \infty\right)$

$= \pi R^2 \left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ...\infty\right) = \pi R^2 \times \frac{1}{1 - \frac{1}{2}} = 2\pi R^2$

<u>Square</u>

sum of areas = $2R^2 + R^2 + \frac{R^2}{2} + \frac{R^2}{4} + ... \infty$

$= R^2 \left(2 + 1 + \frac{1}{2} + \frac{1}{4} + ... \infty\right) = R^2 \frac{2}{1 - \frac{1}{2}} = 4R^2$

Sol.21 **(i)** $\frac{b^2c^2 + c^2a^2}{2} \geq \sqrt{a^2b^2c^4}$

$\Rightarrow \frac{b^2c^2 + c^2a^2}{2} \geq abc^2$ ....(i)

& $\frac{c^2a^2 + a^2b^2}{2} \geq \sqrt{a^4b^2c^2}$

$\Rightarrow \frac{c^2a^2 + a^2b^2}{2} \geq a^2bc$ ....(ii)

& $\frac{a^2b^2 + b^2c^2}{2} \geq \sqrt{a^2b^4c^2}$

$\Rightarrow \frac{a^2b^2 + b^2c^2}{2} \geq ab^2c$ ....(iii)

Add (i), (ii) & (iii)

$b^2c^2 + c^2a^2 + a^2b^2 \geq a^2bc + ab^2c + abc^2$

$\Rightarrow b^2c^2 + c^2a^2 + a^2b^2 \geq abc\,(a + b + c)$

(ii) A.M. > G.M.

$\frac{(a+b-c)+(c+a-b)+(b+c-a)}{3}$

$> ((a + b - c)\,(c + a - b\,(b + c - a))^{1/3}$

$\Rightarrow a + b + c > 3\,[(a + b - c)\,(c + a - b)\,(b + c - a)]$

$\Rightarrow (a + b + c)^3 > 27(a + b - c)\,(c + a - b)(b + c - a)$

Sol.22 **(i)** $\sum_{r=1}^{n} r(r+1)(r+2)(r+3)$

$= \sum_{r=1}^{n} \frac{1}{5} r(r+1)(r+2)(r+3)[(r+4) - (r-1)]$

$= \frac{1}{5} \sum \{r(r+1)(r+2)(r+3)(r+4)$

$-(r - 1)\,(r)\,(r + 1)\,(r + 2)\,(r + 3)\}$

$S_n = \frac{1}{5}$ [1.2.3.4.5 – 0.1.2.3.4] +

$+ \frac{1}{5}$ [2.3.4.5.6 – 1.2.3.4.5] +

+ ---------------------------------------

$+ \frac{1}{5} [n(n + 1)\,(n + 2)\,(n + 3)(n + 4)$

$- (n - 1)\,n\,(n + 1)\,(n + 2)\,(n + 3)]$

$S_n = \frac{1}{5}\, n\,(n + 1)\,(n + 2)\,(n + 3)\,(n + 4)$

(ii) $S_n = \frac{n}{1.2.3} + \frac{n-1}{2.3.4} + ... + \frac{1}{n(n+1)(n+2)}$

$S_n = \Sigma T_n = \sum_{r=1}^{n} \frac{n-r+1}{r(r+1)(r+2)}$

$\Sigma T_n = \sum_{r=1}^{n} \frac{(n+3)-(r+2)}{r(r+1)(r+2)}$

$\Sigma T_n = \sum_{r=1}^{n} \frac{n+3}{r(r+1)(r+2)} - \sum_{r=1}^{n} \frac{1}{r(r+1)}$ ...(i)

$\Sigma T_n = \frac{(n+3)}{2} \sum_{r=1}^{n} \frac{(r+2)-r}{r(r+1)(r+2)} - \sum_{r=1}^{n} \frac{1}{r(r+1)}$

$\Sigma T_n = \left\{\frac{(n+3)}{2} - 1\right\} \sum_{r=1}^{n} \frac{1}{r(r+1)} - \frac{(n+3)}{2} \sum_{r=1}^{n} \frac{1}{(r+1)(r+2)}$

$$\Sigma T_n = \frac{(n+1)}{2}\left[1-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\ldots+\frac{1}{n}-\frac{1}{(n+1)}\right]$$

$$-\frac{(n+3)}{2}\left[\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\frac{1}{4}+\ldots+\frac{1}{(n+1)}-\frac{1}{(n+2)}\right]$$

$$\Sigma T_n = \frac{(n+1)}{2}\left(1-\frac{1}{(n+1)}\right)-\frac{(n+3)}{2}\left[\frac{1}{2}-\frac{1}{(n+2)}\right]$$

$$= \frac{(n+1)}{2}\frac{(n)}{(n+1)} - \frac{(n+3)}{2}\frac{(n)}{2(n+2)}$$

$$\Sigma T_n = \frac{n}{2} - \frac{n(n+3)}{4(n+2)} = \frac{n}{4}\left[\frac{2n+4-n-3}{(n+2)}\right]$$

$$= \frac{n}{4}\frac{(n+1)}{(n+2)}$$

Sol.23 $S = 1^2 - \frac{2^2}{5}+\frac{3^2}{5^2}-\frac{4^2}{5^3}+\frac{5^2}{5^4}-\frac{6^2}{5^5}+\ldots\infty$

$$\frac{S}{5} = \quad \frac{1^2}{5}-\frac{2^2}{5^2}+\frac{3^2}{5^3}-\frac{4^2}{5^4}+\frac{5^2}{5^5}\ \ldots.\infty$$

$$\frac{6}{5}S = 1^2-\frac{(2^2-1^2)}{5}+\frac{(3^2-2^2)}{5^2}-\frac{(4^2-3^2)}{5^3}+\frac{(5^2-4^2)}{5^4}\ldots\infty$$

$$\frac{6}{5}S = 1^2 - \frac{3}{5}+\frac{5}{5^2}-\frac{7}{5^3}+\frac{9}{5^4}\ \ldots.\infty$$

$$\frac{6}{5}S = 1 - \frac{3}{5}+\frac{5}{5^2}-\frac{7}{5^3}\ \ldots.\infty$$

$$\frac{6}{25}S = \quad 1-\frac{3}{5^2}+\frac{5}{5^3}-\frac{7}{5^4}+\ldots\infty$$

$$\Rightarrow \frac{36}{25}S = 1-\frac{2}{5}+\frac{2}{5^2}-\frac{2}{5^3}+\frac{2}{5^4}\ \ldots\infty$$

$$\Rightarrow \frac{36}{25}S = 1-\frac{2}{5}\left(1-\frac{1}{5}+\frac{1}{5^2}-\frac{1}{5^3}+\ldots\infty\right)$$

$$\Rightarrow \frac{36}{25}S = 1-\frac{2}{5}\ \frac{1}{1-\left(-\frac{1}{5}\right)} = 1-\frac{2}{5}\times\frac{5}{6}$$

$$\Rightarrow \frac{36}{25}S = \frac{2}{3} \quad\Rightarrow\quad S = \frac{2\times 25}{3\times 36} = \frac{25}{54}$$

Sol.24 $T_1 = a;\ S_p = 0$

$$\frac{P}{2}\ [2a+(p-1)\,d] = 0$$

$$\Rightarrow 2a+(p-1)\,d = 0 \Rightarrow d = \frac{-2a}{p-1}$$

Let $S = T_{p+1} + T_{p+2} + \ldots + T_{p+q}$

$$S = \frac{q}{2}\ [(a+pd)+(a+(p+q-1)\,d]$$

$$= \frac{q}{2}\ [a+pd+a+pd+qd-d]$$

$$= \frac{q}{2}\ [\{2a+(p-1)\,d\}+(p+q)\,d]$$

$$= \frac{q}{2}\ (p+q)\,d = \frac{q(p+q)(-2a)}{2(p-1)} = \frac{-aq(p+q)}{p-1}$$

Sol.25 Let total no. of terms is 2n

i.e. n terms is even & n terms is odd

Sum of odd terms = $T_1 + T_3 + T_5 + \ldots. + T_{2n-1}$

$$= \frac{n}{2}\ [2a+(n-1)2d] = n\,(a+(n-1)\,d) = 24$$

& sum of even terms = $T_2 + T_4 + T_6 + \ldots + T_{2n}$

$$= \frac{n}{2}[2(a+d)+(n-1)\,2d]$$

$= n\,[a+d+(n-1)\,d] = 30$

$= n\,(a+(n-1)\,d) + nd = 30$

$\Rightarrow 24 + nd = 30 \quad\Rightarrow\quad nd = 6$

$$T_{2n} - T_1 = 10\frac{1}{2} \quad\Rightarrow\ a+(2n-1)\,d-a = \frac{21}{2}$$

$$\Rightarrow 2nd - d = \frac{21}{2} \Rightarrow d = 12-\frac{21}{2} \Rightarrow d = \frac{3}{2}$$

$$n = \frac{6}{d} = \frac{6}{3}\times 2 = 4$$

$n = 4 \qquad\qquad \therefore\ 2n = 8$

Sol.26 $S_{40} = 3600$

$$S_{30} = 3600 - \frac{1}{3}\times(3600) = 2400$$

$$\Rightarrow \frac{40}{2}\ [2a+39d] = 3600 \Rightarrow 2a+39d = 180$$

& $\frac{30}{2}$ [2a + 29d] = 2400

$\Rightarrow$ 2a + 29d = 160

d = 2 & a = 51

$\therefore$ First Installment = 51 Rs.

Sol.27 $T_p = A + (p-1)d = a$ ...(i)

$T_q = A + (q-1)d = b$ ...(ii)

$T_r = A + (r-1)d = c$ ...(iii)

(ii) – (iii) $\Rightarrow (q-r)d = b-c$

$\Rightarrow a(q-r) = \frac{a(b-c)}{d}$

(iii) – (i) $\Rightarrow (r-p)d = c-a \Rightarrow b(r-p) = \frac{b(c-a)}{d}$

(i)–(ii) $\Rightarrow (p-q)d = a-b \Rightarrow c(p-q) = \frac{c(a-b)}{d}$

add. $a(q-r) + b(r-p) + c(p-q) = 0$

Sol.28 $b = \frac{2ac}{a+c} \Rightarrow ac = \frac{b(a+c)}{2}$

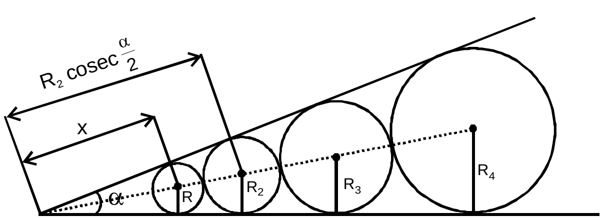
$$\frac{1}{(b-a)} + \frac{1}{(b-c)} = \frac{1}{a} + \frac{1}{c}$$

$$\Rightarrow \text{L.H.S.} = \frac{2b-(a+c)}{b^2-b(a+c)+ac}$$

$$= \frac{2(2b-(a+c))}{2b^2-2b(a+c)+b(a+c)} = \frac{2(2b-(a+c))}{2b^2-b(a+c)}$$

$$= \frac{2(2b-(a+c))}{b(2b-(a+c))} = \frac{2}{b} = \frac{a+c}{ac} = \frac{1}{a} + \frac{1}{c} = \text{R.H.S.}$$

Sol.29 $\frac{R}{x} = \sin\frac{\alpha}{2} \Rightarrow x = R\,\text{cosec}\,\frac{\alpha}{2}$



by similar triangle

$$\frac{R_2}{R_2\,\text{cosec}\,\frac{\alpha}{2}} = \frac{R}{R\,\text{cosec}\,\frac{\alpha}{2}}$$

$\because R_2\text{cosec}\,\frac{\alpha}{2} = R\text{cosec}\,\frac{\alpha}{2} + R + R_2$

$$\Rightarrow \frac{R_2}{R\,\text{cosec}\,\frac{\alpha}{2}+R+R_2} = \frac{R}{R\,\text{cosec}\,\frac{\alpha}{2}}$$

$$\Rightarrow R_2\text{cosec}\,\frac{\alpha}{2} = R\,\text{cosec}\,\frac{\alpha}{2} + R + R_2$$

$$\Rightarrow R_2\left(\text{cosec}\,\frac{\alpha}{2}-1\right) = R\left(\text{cosec}\,\frac{\alpha}{2}+1\right)$$

$$\Rightarrow R_2 = \frac{R\left(1+\sin\frac{\alpha}{2}\right)}{\left(1-\sin\frac{\alpha}{2}\right)}$$

Similarly

$$R_3 = R_2\left(\frac{1+\sin\frac{\alpha}{2}}{1-\sin\frac{\alpha}{2}}\right) \Rightarrow R_3 = R\left(\frac{1+\sin\frac{\alpha}{2}}{1-\sin\frac{\alpha}{2}}\right)^2$$

sum of first 'n' radius = $R + R_2 + R_3 + \ldots R_n$

$$= \frac{R\left\{\left(\frac{1+\sin\frac{\alpha}{2}}{1-\sin\frac{\alpha}{2}}\right)^n - 1\right\}}{\left(\frac{1+\sin\frac{\alpha}{2}}{1-\sin\frac{\alpha}{2}}\right)-1} = \frac{R\left(1-\sin\frac{\alpha}{2}\right)}{2\sin\frac{\alpha}{2}}\left\{\left(\frac{1+\sin\frac{\alpha}{2}}{1-\sin\frac{\alpha}{2}}\right)^n - 1\right\}$$

Sol.30 a =1

$S_9 = 369 = \frac{9}{2}$ [2 + 8.d]

$\Rightarrow$ 82 – 2 = 8d $\Rightarrow$ d = 10

First term of G.P. = a = 1

9^{th} term of G.P. = $a.r^8 = a + 8d$

$r^8 = 1 + 80 \quad \Rightarrow r^8 = 81$

$\Rightarrow r^8 = 3^4 \quad \Rightarrow r = \sqrt{3}$

7^{th} term = $ar^6 = 1 . (\sqrt{3})^6 = 3^3 = 27$